C-10

Dimensional Analysis:→

102/510 **Buckingham's Π theorem:→**

$$\phi(Q_1, Q_2, Q_3, \ldots Q_n) = 0$$

Where $Q_1, Q_2, Q_3, \ldots\ldots Q_n$ are dimensinal Physical Quantities then there can be $(n-m)$ dimensionless Π quantities that describe the same phenomenon as

$$f[\Pi_1, \Pi_2, \Pi_3, \ldots\ldots \Pi_{n-m}] = 0$$

Proof:-

The no. of dimensionless required to sepecify completly the relationship is the number of variable n, the number of dimensions m involved in the variables.

Consider $\phi(Q_1, Q_2, Q_3, Q_4, Q_5)$

$$\phi(Q_1, Q_2, Q_3, Q_4, Q_5) = 0 \quad \text{—(1)}$$

(1) Expressed form of a Powerseries

$$\phi(Q_1, Q_2, Q_3, Q_4, Q_5) = A_1(Q_1^\alpha, Q_2^\beta, Q_3^\gamma, Q_4^\delta, Q_5^\epsilon)$$
$$+ A_2(Q_1^\alpha, Q_2^\beta, Q_3^\gamma, Q_4^\delta, Q_5^\epsilon)$$
$$\ldots\ldots + A_K(Q_1^\alpha, Q_2^\beta, Q_3^\gamma, Q_4^\delta, Q_5^\epsilon) \quad \text{—(2)}$$

Where A_K's are dimensionless coefficient

And the exponents $\alpha, \beta, \gamma, \delta, \epsilon$ are integer

~~detr~~ All term certain the same dimensions

$$(Q_1^\alpha Q_2^\beta Q_3^\gamma Q_4^\delta Q_5^\epsilon)^k = M^{ka} L^{kb} T^{kc}$$

For equal dimensions

$$M^a L^b T^c = M^{ka} L^{kb} T^{kc}$$

$$\Rightarrow a = b = c = 0, \text{ for } k \neq 0$$

it follows that every term (2) must be dimensionless.

$$(Q_1^\alpha Q_2^\beta Q_3^\gamma Q_4^\delta Q_5^\epsilon) = M^a L^b T^c \quad \text{has zero dimen.} \quad \text{—(3)}$$

$Q_1 = M^{a_1} L^{b_1} T^{c_1}$

$Q_2 = M^{a_2} L^{b_2} T^{c_2}$

$Q_5 = M^{a_5} L^{b_5} T^{c_5}$ —— (4)

From (3) And (4) wehave

$$(M^{a_1}L^{b_1}T^{c_1})^{\alpha}(M^{a_2}L^{b_2}T^{c_2})^{\beta}(M^{a_3}L^{b_3}T^{c_3})^{\gamma}(M^{a_4}L^{b_4}T^{c_4})^{\delta}(M^{a_5}L^{b_5}T^{c_5})^{\epsilon} = M^0L^0T^0$$

Equating the Power of M, L, T wehave

$$a_1\alpha + a_2\beta + a_3\gamma + a_4\delta + a_5\epsilon = 0 \quad \text{—— (5)}$$

$$b_1\alpha + b_2\beta + b_3\gamma + b_4\delta + b_5\epsilon = 0 \quad \text{—— (6)}$$

$$c_1\alpha + c_2\beta + c_3\gamma + c_4\delta + c_5\epsilon = 0 \quad \text{—— (7)}$$

Since there are only three equations, we may solve three exponents α, β, γ in terms of the others.

$$a_1\alpha + a_2\beta + a_3\gamma = -a_4\delta - a_5\epsilon \quad \text{—— (8)}$$

$$b_1\alpha + b_2\beta + b_3\gamma = -b_4\delta - b_5\epsilon \quad \text{—— (9)}$$

$$c_1\alpha + c_2\beta + c_3\gamma = -c_4\delta - c_5\epsilon \quad \text{—— (10)}$$

Now α, β, γ canbe determinant in term of the other δ, ϵ

solving Equation (8) (9) (10) wehave

$$\boxed{\gamma = p_3\delta + q_3\epsilon}$$

similarly other two exponents writtenas

$$\boxed{\alpha = p_1\delta + q_1\epsilon}$$

And $$\boxed{\beta = p_2\delta + q_2\epsilon}$$

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10.3 Similitude:→

Two flows are said to be mechanically Similar. they are
(i) Kinematically
(ii) Dynamically
(iii) Geometrically
two flows are said to be geometrically similar, if they are canbe made to appear Photographically alike. i.e. they differ in their Absolute size.
The ratios of corresponding length dimensions between the model and proto type must be Same. the ratios of characteristic length $\frac{l_1}{l_2}$ is a constant at corresponding parts of two flow fields.
Kinematic Similarity shows Similarity of moti on. the ratios of the velocity and acceleration of two flows must be same at the corresponding Point. the magnitude and direction of velocity and acceleration at the corresponding Point in the two flows, should be the same. the stream line will therefore, appear, similar two flows. we have

$$\frac{q}{q'} = \frac{q_p}{q'_p}$$

And

$$\frac{f}{f'} = \frac{f_p}{f'_p}$$

Geometric similarity is a necessary condi-tion but not a sufficient condition for Kinematic Similarity.

Dynamic Similarity:- It is a

consider the Equation of motion for the two flows.

$$\rho_1 \frac{dq_1}{dt_1} = \rho_1 q_1 - \nabla_1 p_1 + \mu_1 \nabla_1^2 q_1$$

$$\rho_2 \frac{dq_2}{dt_2} = \rho_2 q_2 - \nabla_2 p_2 + \mu_2 \nabla_2^2 q_2$$

For the flows to be dynamically similar. the corresponding ratio

$$\frac{\rho_2 \frac{dq_2}{dt_2}}{\rho_1 \frac{dq_1}{dt_1}} = \frac{\rho_2 q_2}{\rho_1 q_1} = \frac{\nabla_2 p_2}{\nabla_1 p_1} = \frac{\mu_2 \nabla_2^2 q_2}{\mu_1 \nabla_1^2 q_1}$$

(i) (ii) (iii) (iv)

I **Froude Number:-** we know

$$\frac{\rho_2 \frac{dq_2}{dt_2}}{\rho_1 \frac{dq_1}{dt_1}} = \frac{\rho_2 q_2}{\rho_1 q_1}$$

$$\Rightarrow \frac{\rho_2 \frac{dq_2}{dt_2}}{\rho_2 q_2} = \frac{\rho_1 \frac{dq_1}{dt_1}}{\rho_1 q_1}$$

$$\Rightarrow \frac{\cancel{\rho_2} \left(\frac{U_2}{L_2 / U_2}\right)}{\cancel{\rho_2} q_2} = \frac{\cancel{\rho_1} \left(\frac{U_1}{L_1 / U_1}\right)}{\rho_1 q_1}$$

$$\Rightarrow \boxed{\frac{U_2^2}{L_2 q_2} = \frac{U_1^2}{L_1 q_1} = F_r}$$

F_r know as Froude number

Physically this represent the rate of change of Velocity at each Point depending on the initial boundary conditions.

For variable density flows.

$$\frac{\text{Material Velocity}}{\text{Sound Velocity}} = \frac{U_1}{a_1} = \frac{U_2}{a_2} = M$$

where M is known as mach Number.
Physically this define number of the compressibility of the fluid due to high speed, when the mach number is nearly one or greater than one, the fluid will be taken as compressible

Reynolds number:- The Reynolds number Re is

$$Re = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

we know

$$\Rightarrow \frac{\rho_2 (dq_2/dt_2)}{\rho_1 (dq_1/dt_1)} = \frac{\mu_2 \nabla_2^2 q_2}{\mu_1 \nabla_1^2 q_1}$$

$$\Rightarrow \frac{\rho_2 (dq_2/dt_2)}{\mu_2 \nabla_2^2 q_2} = \frac{\rho_1 (dq_1/dt_1)}{\mu_1 \nabla_1^2 q_1}$$

$$\Rightarrow \frac{\rho_2 U_2 / (l_2/U_2)}{\mu_2 U_2 / l_2^2} = \frac{\rho_1 U_1 / (l_1/U_1)}{\mu_1 U_1 / l_1^2}$$

$$\Rightarrow \frac{\rho_2 U_2^2 / l_2}{\mu_2 U_2 / l_2^2} = \frac{\rho_1 U_1^2 / l_1}{\mu_1 U_1 / l_1^2}$$

$$\Rightarrow \frac{\rho_2 U_2 l_2}{\mu_2} = \frac{\rho_1 U_1 l_1}{\mu_1} = Re$$

which is called Reynolds number

$$\boxed{Re = \frac{UL}{\nu}}$$

Reynolds no is a measure of the relative importance of dynamic forces and viscous forces